

# Modeling retinal high and low contrast sensitivity filters

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## Abstract

In this paper two types of ganglion cells in the visual system of mammals (monkey) are modeled. A high contrast sensitive type, the so called M-cells, which project to the two magno-cellular layers of the lateral geniculate nucleus (LGN) and a low sensitive type, the P-cells, which project to the four parvo-cellular layers of the LGN. The results will be compared with the ganglion cells as described by Kuffler.

## 1 Introduction

In the last forty years a lot of work on the mammalian visual system has been done by neuro-physiologists. It was Kuffler [1] who in 1952 recorded the activity from the axons of the retinal ganglion cells that make up the optic nerve. His experiments revealed the type of receptive fields a retinal ganglion cell possess. He found two basic types of center-surround cells. Later Hubel [2] and Wiesel did a lot of pioneering work in the primary visual cortex, its is mainly due to their work that today we understand the functionality of certain parts in the visual system of mammals.

There are two types of ganglion cells that will be modeled in this paper. The first type of cells are the *ganglion magnocellular cells* or short the *M-cells*, this type of cells are retinal ganglion cells which project to the magnocellular layers (ventral layers) in the lateral geniculate nucleus. The second type of cells are *parvocellular cells* or *P-cells* which project to the parvocellular layers (dorsal layers) in the lateral geniculate nucleus.

The two ventral layers in the lateral geniculate nucleus are more sensitive to luminance contrast than the cells in the four dorsal layers of the lateral geniculate nucleus. The differences in these layers is due to the fact that there are differences in the retinal ganglion cells which provide excitatory synaptic input to the LGN-neurons and not because of the organization (pattern of connectivity) in the LGN [3].

The small P-cells in the monkey are color sensitive (wavelength selective) and have small concentric center-surround receptive fields. They are not very contrast sensitive. The large M-cells are not wavelength selective, they also have concentric center-surround receptive fields but these receptive fields are larger than the receptive fields of the P-cells and are highly sensitive to contrast [4, 5, 6].

In section 2 the center-surround receptive field will be modeled by a so called *mexican-hat function*. The function can be modeled by taking the difference of two gaussian functions (DOGs). In this section is also described how the center and surround of the mexican-hat function can be controlled by these two gaussians. In this paper the receptive fields of M-cells and P-cells modeled with the mexican-hat function are only used for comparison. The center and surround of the mexican-hat function in fact are used to calculate respectively the average center and surround luminance in the receptive field of an M-cell or P-cell. This implies that a mexican-hat filter is used which is constant in both center and surround but differs in sign, i.e. the center of the mexican-hat is positive and surround is negative. In section 3 the relation between dendritic tree and visual field is given. In the fourth section a contrast filter for the two types of cells is created and after that the receptive fields of an M-cell and P-cell are modeled. In the last section the experimental results are given.

## 2 Center-surround receptive fields

In this section the properties of gaussians are described which are used for modeling a center-surround receptive field.

The standard two-dimensional mexican-hat function which is derived from one gaussian function can be defined as follows:

$$G_m(r) = \frac{2\sigma - r^2}{\sigma^2} e^{-\frac{r^2}{2\sigma}} \quad (1)$$

where  $r = \sqrt{x^2 + y^2}$ . The relation between the center and surround radius is  $1 : \sqrt{-\log \varepsilon}$  where  $\varepsilon$  is a positive constant near zero. If  $\varepsilon = e^{-9}$  then we get a ratio of  $1 : 3$ . This means in fact that one is only able to control either center or surround of a mexican-hat function, for details of (1) and its properties we refer to [7].

## 2.1 The mexican-hat function with differences of two gaussians

In case one wants to be able to control both center and surround, a mexican-hat function should be modeled with two gaussians. The difference of these two gaussians (DOGs) will give the desired mexican-hat function.

Let us define the *center* of the mexican-hat as that part of the the function for which  $G_m(r) \geq 0$  and the *surround* as the parts for which  $G_m(r) < 0$ . The mexican-hat function can be created by taking the difference between a center and a surround gaussian. Assume that the mexican-hat is modeled by a center gaussian  $G_c$  which is subtracted from a surround gaussian  $G_s$ , where the center gaussian is defined as:

$$G_c(r) = e^{-cr^2} \quad (2)$$

and the normalized surround gaussian is defined by:

$$G_s(r) = \frac{1}{m^2} e^{-c \frac{r^2}{m^2}} \quad (3)$$

where  $\frac{1}{m^2}$  is the normalization-factor,  $m > 1$ . The ratio between center and surround gaussian is  $1 : m$ . The mexican-hat function is a combination of the two previous equations:

$$G_m(r) = G_c(r) - G_s(r) \quad (4)$$

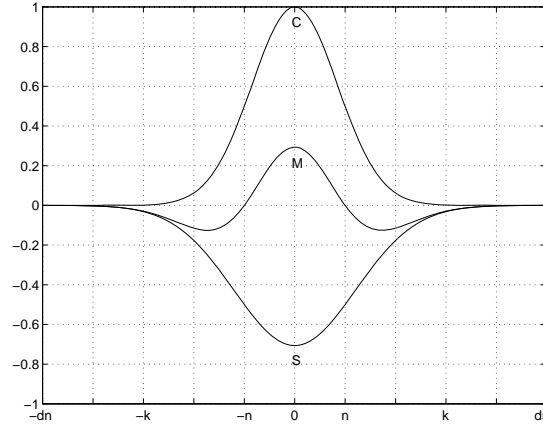


Figure 1: The center gaussian C, surround gaussian S, and the mexican hat function M. For better visualization the one-dimensional functions are used.

For normalization of two-dimensional gaussian filters the following integral is used:

$$\iint e^{-\gamma(x^2+y^2)} dx dy = \frac{\pi}{\gamma} \quad (5)$$

where  $\gamma$  is a constant. This implies that the appropriate normalization factor is  $\gamma$ .

The border of a gaussian is used to control the center or surround of a mexican-hat function. The border of the center gaussian is defined as the radius  $k$  for which  $G_c(k) = \varepsilon$  (Figure 1). Since a gaussian will never be exactly 0, a small value  $\varepsilon > 0$  will be used to estimate the border of a gaussian:

$$G_c(k) = \varepsilon \quad (6)$$

$$\begin{aligned} \equiv \\ c &= -\frac{\log \varepsilon}{k^2} \end{aligned} \quad (7)$$

## 2.2 Controlling the difference of two gaussians

Let us assume that the center of the mexican-hat function has radius  $n$  and that the surround has a radius that is related to the center radius in such a way that the surround radius is  $d$  times larger than the center radius (see also

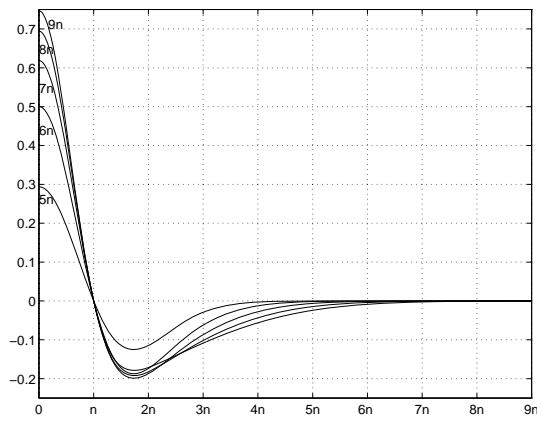


Figure 2: Five different mexican-hat functions all with the same center radius  $n$  but different surround radii, the different surround radii are  $5n$ ,  $6n$ ,  $7n$ ,  $8n$ , and  $9n$ , respectively. For better visualization again the one-dimensional functions are used.

Figure 1). Given the center and surround gaussians from (2) and (3), we should find out how  $c$  and  $m$  should be chosen to get a mexican-hat center radius  $n$  and a surround radius  $dn$  (Figure 2).

For the center radius of the mexican-hat function we obtain:

$$G_m(n) = 0 \quad (8)$$

$$\equiv c = \frac{-2 \log m}{n^2 \left( \frac{1}{m^2} - 1 \right)}, \quad (9)$$

from which we conclude that there is a relation between  $c$  and  $m$ .

For the surround radius of the mexican-hat function we get:

$$G_m(dn) = -\varepsilon \quad (10)$$

$$\equiv d = \sqrt{\frac{\log(m^2 \varepsilon) (1 - m^2)}{2 \log m}}. \quad (11)$$

Where  $\varepsilon > 0$  is a small constant. Equation (11) gives the relation between  $m$  and  $d$ .

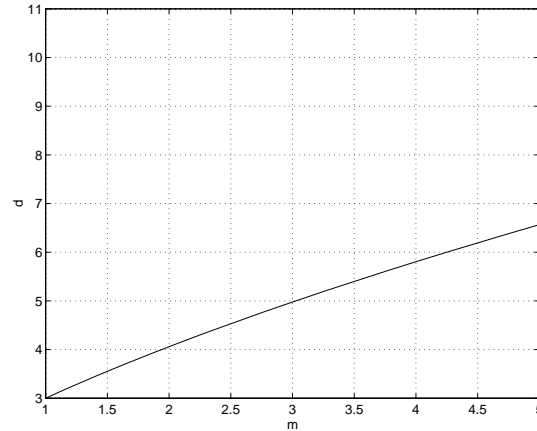


Figure 3: The relation between  $m$  and  $d$  in the two-dimensional case. Note that the minimum  $d = \sqrt{-\log \varepsilon}$ . This means that only a relation between center and surround can be realized where the surround radius is at least  $\sqrt{-\log \varepsilon}$  times larger than the center radius.

### 3 The relation between receptive field and dendritic tree of a ganglion cell

The direct connections from a ganglion cell to the bipolar cells is called the dendritic field of a ganglion cell and the maximum size or radius of such a dendritic field is the dendritic field size or radius of a ganglion cell. The dendritic field size of a ganglion cell to the receptors is defined as the maximum dendritic field size that can be reached by the direct pathway, from receptors to bipolars to ganglion cells [2], and is shown in Figure 4.

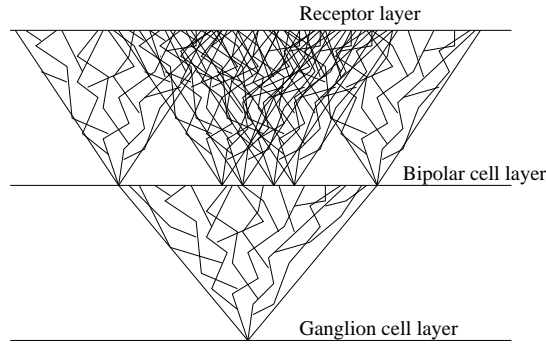


Figure 4: The dendritic field of the ganglion cell itself plus the dendritic fields of the bipolar cells which are directly connected with this ganglion cell is defined as the dendritic field of the ganglion cell to the receptors.

The direct pathway is responsible for the the receptive field centers. The indirect pathway, the path also via the horizontal cells and amacrine cells, is responsible for the the surround response of the receptive field. From this we conclude that the dendritic field radius of a ganglion cell to its receptors is equal to the receptive field center radius of the same ganglion cell. This implies that the variation of the dendritic field size with retinal eccentricity of the cells is equal to the variation in receptive field center size of the cells.

In [4] the dendritic field sizes of the ganglion cells grow in a linear way with retinal eccentricity. We also assume that the dendritic field sizes of the bipolar cells grow in a linear way with increasing retinal eccentricity. It can be easily verified that the dendritic tree between ganglion cell and receptors also grows in a linear way with increasing retinal eccentricity.

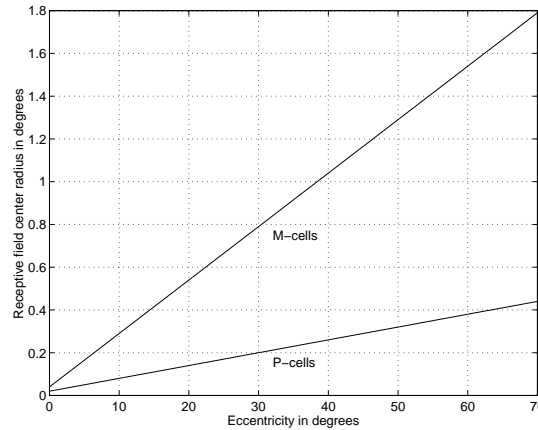


Figure 5: The receptive field center radius grows linear with eccentricity for both M-cells and P-cells. Note that the receptive fields of the M-cells are about a factor four larger than the P-cells.

In the model the assumption is made that given the eccentricity both ganglion cells and bipolar cells have exactly the same dendritic field radius. In [4] the dendritic field size for the ganglion cells to the bipolar cells with retinal eccentricity is given, in the model given the assumption that both ganglion and bipolar cells have the same dendritic field radius then the dendritic field radius from ganglion cells to the receptors is twice the size of the dendritic field radius of the ganglion cells. The receptive field center radius for a given eccentricity  $\alpha$  for the M-cells (Figure 5) will be:

$$M_c(\alpha) = 2.5 \cdot 10^{-2} \alpha + 4.0 \cdot 10^{-2} \quad (12)$$

and the smaller receptive field center radius of the P-cells (Figure 5) will be:

$$P_c(\alpha) = 6.0 \cdot 10^{-3}\alpha + 4.0 \cdot 10^{-2} \quad (13)$$

where  $\alpha$  is the eccentricity in degrees.

## 4 Modeling M-cells and P-cells

Instead of using the mexican-hat as a filter for modeling the receptive field of an M-cell or a P-cell, the center and surround of the mexican-hat function are used to calculate the average center and surround luminance in a receptive field respectively. This implies that a mexican-hat filter is used which is constant in both center and surround but differs in sign, i.e. the center of the mexican-hat is positive and surround is negative. Note that the M-cells and P-cells are also modeled with the mexican-hat function but that it is only used for comparison. (For the center-surround (mexican-hat) filter see [7]). In fact the filter uses only the center radius  $n$  and the surround radius  $dn$ , which is chosen to be about three times larger than the center radius of the mexican-hat function.

If the center of a mexican-hat is placed at position  $(\varphi, r)$  in the visual field then the luminance of the center  $L_c$  is defined as the average luminance of that part of the visual field which fits in the center of the mexican-hat:

$$L_c(\varphi, r) = \frac{\int_{\varphi_1=0}^{2\pi} \int_{r_1=0}^n I(r \cos \varphi + r_1 \cos \varphi_1, r \sin \varphi + r_1 \sin \varphi_1) r_1 dr_1 d\varphi_1}{\int_{\varphi_1=0}^{2\pi} \int_{r_1=0}^n r_1 dr_1 d\varphi_1} \quad (14)$$

Note that  $n$  is the center radius, for the M-cell  $n = M_c(\alpha)$  and for the P-cell  $n = P_c(\alpha)$ . For the artificial visual field  $I$  a two-dimensional image is used.

The luminance of the surround  $L_s$  is defined as the average energy in the visual field which fit in the surround of the mexican-hat:

$$L_s(\varphi, r) = \frac{\int_{\varphi_1=0}^{2\pi} \int_{r_1=n}^{dn} I(r \cos \varphi + r_1 \cos \varphi_1, r \sin \varphi + r_1 \sin \varphi_1) r_1 dr_1 d\varphi_1}{\int_{\varphi_1=0}^{2\pi} \int_{r_1=n}^{dn} r_1 dr_1 d\varphi_1} \quad (15)$$

where  $d$  is about 3, which implies that the center is about three times smaller than the surround.

Because both M-cells and P-cells are contrast sensitive, we introduce the term *contrast*. Contrast is the relative difference between the maximum and minimum luminance and is defined as follows:

$$\text{Contrast} = \frac{L_{\max} - L_{\min}}{L_{\max} + L_{\min}} \quad (16)$$

where  $L_{\max}$  and  $L_{\min}$  are the maximum and minimum luminances respectively.

From the receptive field sizes (12)-(13) and the definition of contrast (16) the *contrast filter* is rather trivial, it is the contrast between  $L_c$  and  $L_s$ , given a position  $(\varphi, r)$  in the visual field:

$$C(\varphi, r) = \frac{|L_c(\varphi, r) - L_s(\varphi, r)|}{L_c(\varphi, r) + L_s(\varphi, r)} \quad (17)$$

Note that the size of the center,  $M_c$  and  $P_c$  of the M-cell and P-cell respectively, gives the difference between a contrast filter for the M-cell or P-cell.

The response of the M-cell or P-cell depends on the contrast. The contrast-response for the M-cell, which is defined as the receptive field of the M-cell or high-contrast sensitive filter is as follows:

$$R_M(C_M) = \frac{aC_M}{0.13 + C_M} \quad (18)$$

where  $C_M$  is the contrast difference for an M-cell and  $a$  is the impulse amplitude. For the P-cells the contrast-response is defined as the receptive field of the P-cell or low-contrast sensitive filter and is as follows:

$$R_P(C_P) = \frac{aC_P}{1.74 + C_P} \quad (19)$$

where  $C_P$  is the contrast differences for a P-cell and  $a$  is the impulse amplitude which is identical to the impulse amplitude of the M-cell. Both contrast-response equations are taken from [3].

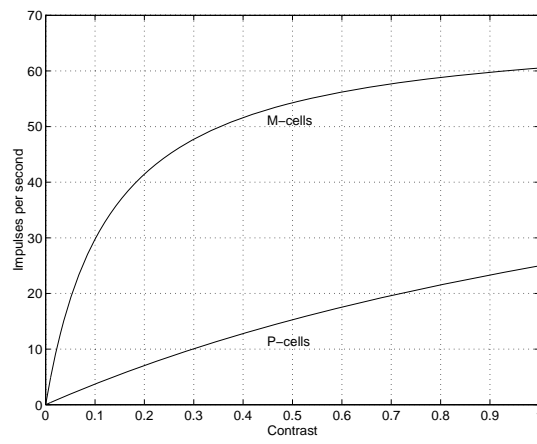


Figure 6: For both type of cells a response-contrast function is given. The magnocellular cells (M-cells) give a high response amplitude on stimulus contrast. This in contrast with the parvocellular cells (P-cells) which give a lower contrast response.

## 5 Experimental results and future research

The results filtering the face image of Figure 7 with a mexican-hat filter for M-cells and P-cells are shown in Figure 8a and 8b respectively. If we compare the results of the mexican-hat filter (Figure 8a-b) with the contrast filter (Figure 8c-d), it is clear that the results obtained by the contrast filter are better. For example compare the eyebrow of Figure 8a, which is almost invisible, with the eyebrow in Figure 8c. It is remarkable that the differences between Figure 8c and Figure 8e are really small, this means that the contrast filter of a P-cell and the receptive field of the same P-cell have identical properties.

In future research we want to create a model that is able to detect and recognize objects in a visual scene by applying techniques which are also used by mammals. In this model the wavelength selective P-cells give high detailed information which will be used for both color and detailed information description of an object. The non-color selective M-cells which are large will be used together with the simple cells [8, 9] for edge detection and partly for object detection. By using these biologically motivated techniques we hope to get more insights in the functionality of the human brain.



Figure 7: An artificial visual field, which is represented by a face image.

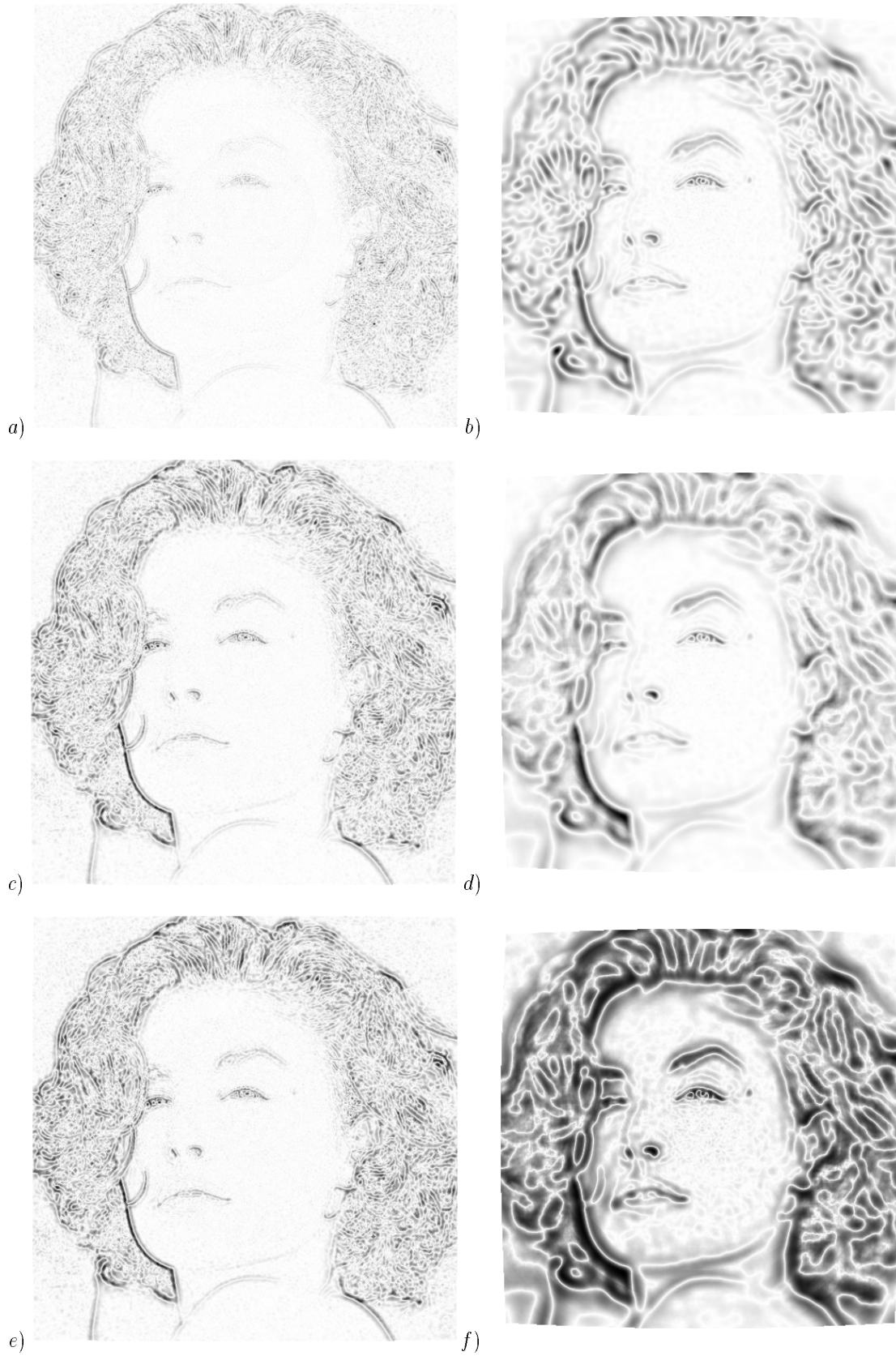


Figure 8: The three left images are related to the parvocellular properties. The three images on the right are related to the magnocellular properties. The original image filtered with a mexican-hat filter *a)* and *b)* . The contrast filters are shown in *c)* and *d)*. The impulse responses on the contrast image are shown in *e)* and *f)*.

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