

# A Model of Spatial Filtering by Visual Cortical Simple Cells

*T. Lourens*

Department of Mathematics and Computer Science  
Rijksuniversiteit Groningen  
Blauwborgje 3  
P.O. Box 800, 9700 AV Groningen  
The Netherlands

T. Lourens. A model of spatial filtering by visual cortical simple cells. In R Moreno-Díaz and J. Mira-Mira, editors, *Brain Processes, Theories, and Models. An International Conference in Honor of W.S. McCulloch 25 Years after his Death*, pages 391–400, Las Palmas de Gran Canaria, Spain, November 12-17 1995. MIT press, Cambridge USA.

# A Model of Spatial Filtering by Visual Cortical Simple Cells

T. Lourens

Department of Computer Science, University of Groningen

P.O. Box 800, 9700 AV Groningen, The Netherlands

E-mail: tino@cs.rug.nl

## Abstract

In this paper a filter is modeled, based on the visual system of mammals. In the filter the retinal ganglion receptive fields and simple cortical receptive fields are used as described by neurophysiologists. The functionality of a ganglion cell and a simple cell are modeled with a gaussian and Gabor function respectively. In the filter also the linear magnification function is included which gives the size of the receptive field with retinal eccentricity. With modeling the filter we hope to get more insights in the human visual system and in the primary visual cortex especially.

## 1 Introduction

In the past forty years much work on sensory areas has been done by neurophysiologists, so parts in the primary visual cortex (of the macaque monkey) are now well understood. Kuffler was the first one who recorded the activity from the axons of the retinal ganglion cells that make up the optic nerve [10]. His experiments revealed the type of *receptive fields* the retinal ganglion cells possess. Kuffler discovered that a retinal ganglion cell or a geniculate cell responds best to a circular spot of light, he found two basic types of cells (the *center-on surround-off* cell and the *center-off surround-on* cell). Later Hubel and Wiesel did a lot of pioneering work in the *primary visual cortex* (also called striate cortex or area 17). They explored various visual cortical regions with their recording micro-electrodes and found two types of cells, which they called *simple* and *complex* cells respectively. The orientation sensitive cells that belong to the first type are one of the most remarkable discoveries [9].

Hubel and Wiesel [8] also described the topographic mapping of the visual fields onto the cortex and the relation between field size and magnification factor. The magnification factor is the relation between a one millimeter movement in the cortex and the displacement in the visual field [7, 2]. The maximum receptive field size of a cell in the primary visual cortex can be derived from the magnification factor and the fact that a two millimeter movement in the cortex is needed to displace fields from one region to an entirely new region. From Fischer [4] we know that the receptive fields of the cats ganglion cells grow in a linear way with retinal eccentricity.

In the paper a comparison between the cortical map and ganglion map is made. For this comparison the assumption is made that both maps only contain center-surround cells and that both, a ganglion and a cortical cell with the same eccentricity (distance from the center of the visual field) have the same receptive field size, although the receptive field of a cell in the cortex has usually not the same size as the receptive field of a ganglion cell.

In our previous work we used the fields with circular symmetry, modeled by a *mexican-hat* (gaussian) function and the orientation sensitive cells, modeled by a *Gabor* (a gaussian multiplied with a sinusoidal) function all separately [13, 14, 12], this in contrast with the the model described in this paper, where the receptive field size depends on the eccentricity and the fields are related to each other.

The paper is organized as follows: In section 2 the relation between the sizes in the cortex in relation with the visual field will be modeled. We will define a visual field angle in relation with a distance (in millimeters) in the cortex. With this relation we are then able, given a visual field position, to calculate the position in the cortex. Section 3 will give a description of the physical structure of a two dimensional

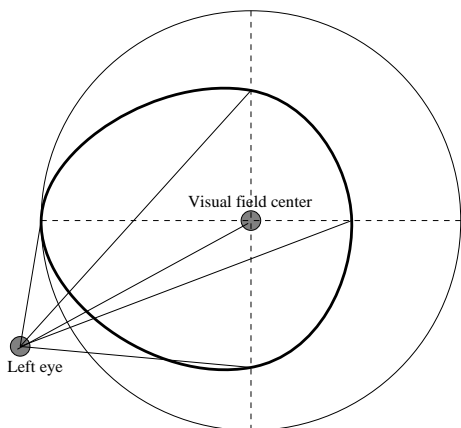
artificial field and its limits. This means that we are going to describe the limits of electronic devices, such as a the lens of a camera and a frame grabber. Section 4 outlines the sizes and types of receptive fields, and how a receptive field can be represented by a mathematical formula. In the fifth section a cortical filter is described. First a local filter is defined, which has a known position and size in the visual field. These local filters are used in the cortical filter. The last section shows the experimental results.

In future research cortical filters of complex and end-stopped will also be modeled. After that the model will be extended and used for object detection and recognition.

## 2 Magnification between cortex and visual field

Before we define the magnification between the primary visual cortex and the visual field, let us first consider the visual fields and the visual range of a human being or another mammal. A human being has two visual fields, one for the left - and one for the right eye. In Figure 1 the visual field for the left eye is shown. In the model we do not take in account the limited physical visual field angles but the maximum visual field angle reached by the horizontal left angle of the left eye, we define the maximum visual field angle as  $VF_{\alpha_{max}}$ .

The magnification is a relation between an angle in the visual field and a size in the primary visual cortex. In the center (fovea) of the visual field a human being has a more detailed image then at the periphery, this suggests that more information from the center of the visual field is (temporary) stored in the primary visual cortex in comparison with the information from the periphery of the visual field. This implies that a larger surface in the cortex is used for a constant surface in the center of the visual field then when the same constant surface is used in the periphery of the visual field. Daniel and Whitteridge [2] gave a good description of the relation between magnification in ( $mm/o$ ) and visual field angle in ( $o$ ). They showed not only that the magnification drops logarithmically if one moves from the center of the visual field to



**Figure 1:** The visual field for the left eye of a human being with a maximum horizontal left angle of about  $70^\circ$ , a maximum horizontal right angle of about  $35^\circ$  due to physical problems, our nose prohibits us from an angle which would be about the same as the horizontal left angle. The vertical upper angle and vertical lower angle are about  $40^\circ$  and  $50^\circ$  degrees respectively. The vertical angles are also both limited due to the fact that the physical structure of the face prohibits a larger angle. In the model only the largest possible visual field angle is used, which implies that one gets a circular visual field.

the magnification drops logarithmically, if one moves from the center to the periphery of the visual field, but also that the inverted magnification shows a logarithmic behavior then the following function L (Figure 2b) can be used:

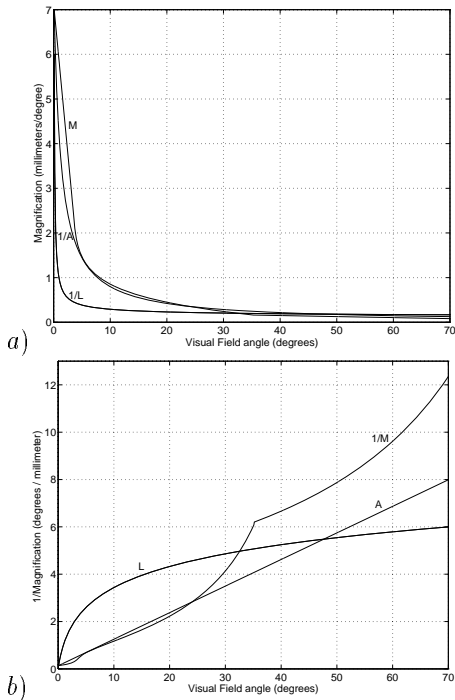
the far periphery, but also that this relation is valid in every direction. Which means that if the angle between the fovea and a certain position in the visual field is constant, that the magnification is also constant, independent of the position in the visual field.

Hubel and Wiesel [8] also show a relation between magnification and eccentricity, in contrast to Daniel and Whitteridge who measured to an eccentricity of about  $65^\circ$ , they measure within a maximum distance angle of  $22^\circ$  from the fovea and give a description of magnification $^{-1}$  in ( $o/mm$ ). Thus they express a movement of one millimeter in the cortex as a displacement of magnification $^{-1}$  degrees in the visual field. If their results are compared with the results from Daniel and Whitteridge, it shows that the results are about the same with respect to the area where in they measured.

If one tries to make a function approximation M (Figure 2a) of the magnification factor from the results reported by [2], one could take a functional approximation which is described by two linear functions and one logarithmic function:

$$\begin{aligned} M(\alpha) &= 7.00 - 1.34\alpha & \alpha \in [0 - 3.5) \\ &= 1.69 - 0.44 \log(\alpha - 3.25) & \alpha \in [3.5 - 35) \\ &= 2.42 \cdot 10^{-1} - 2.3 \cdot 10^{-3}\alpha & \alpha \in [35 - 65) \end{aligned} \quad (1)$$

where  $\alpha$  is the retinal eccentricity or visual field angle and M is the magnification factor. As an inverted magnification is used, as suggested by Hubel, and one assumes that not only



**Figure 2:** The magnification  $M$  (1) and the inverted functions  $L^{-1}$  and  $A^{-1}$  are shown in a). In b) the inverted magnification  $M^{-1}$ , the logarithmic function  $L$  (2), and linear approximation  $A$  (3) are shown.

$VF\alpha_{\max}$ ) 1 millimeter in the cortex corresponds with  $8^\circ$  in the visual field. As one moves from the fovea to the far periphery the relation between cortex and visual field drops logarithmically because its inverted magnification increases in a linear way (3).

Since we are interested in how the visual field is mapped on the cortex, we define the *center of the primary*

*visual cortex* as the position in the primary visual cortex where the center of the visual field is mapped. With this definition we are able to define the relation between the radius from the center of the primary visual cortex and the visual field angle. With a given cortex radius  $\tilde{r}$  (in millimeters) the visual field angle  $\alpha$  is calculated as follows:

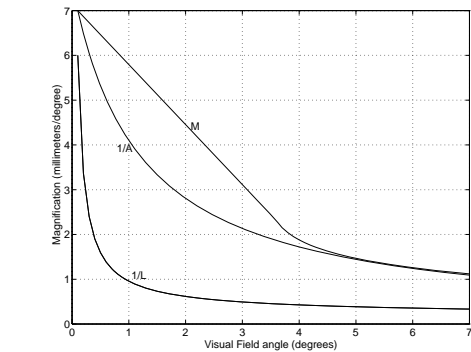
$$\alpha(\tilde{r}) = \frac{x_0}{x}(x+1)^{\tilde{r}} - \frac{x_0}{x} \quad (4)$$

where  $x_0 = 0.14$  and  $x = 0.11$  as defined in the linear inverted magnification (3). Equation (4) is derived from the recursive function  $\alpha(n)$ :

$$\alpha(n) = \alpha(n-1) + A(\alpha(n-1)) \quad (5)$$

Which is equivalent with:

$$\alpha(n) = \frac{x_0}{x}(x+1)^n - \frac{x_0}{x}$$



**Figure 3:** This is an enlargement for the center of the visual field of Figure 2a, to emphasize the difference between the magnification factor  $M$  and the inverted approximated linear increasing function  $A^{-1}$ .

$$L(\alpha) = \frac{1}{6} + c \log(\alpha + 1) \quad \alpha \in [0, VF\alpha_{\max}]. \quad (2)$$

Where  $\alpha$  is the visual field angle and  $c$  is a constant:  
 $c = \frac{35}{6 \log(VF\alpha_{\max} + 1)}$ .

From the inverted magnification  $M^{-1}$  the third linear increasing approximation A (Figure 2b) was derived (See also Fig 6A. from [8] for a linear increasing inverted magnification factor) which is defined as follows:

$$A(\alpha) = 0.14 + 0.11\alpha \quad (3)$$

where  $\alpha$  is the distance from the fovea in degrees and A the magnification<sup>-1</sup> factor. Note that Hubel and Freeman [7] also used a linear magnification<sup>-1</sup> factor. They used  $A(\alpha) = 0.11 + 0.06\alpha$ .

If we consider the inverted function approximation  $M^{-1}$ , we may conclude that the function is linear as estimated by A. Although at first site one may say that there is a big difference if the visual field angle is larger than  $25^\circ$ . This in fact is true but if the approximation A is inverted ( $A^{-1}$ ) it gives a really good approximation as shown in Figure 2a. The difference is not in the periphery as one may expect but near the fovea (i. e. a visual field angle  $< 5^\circ$ ) as shown in Figure 3.

It is clear that the visual fields map systematically onto the cortex. The relation between visual field and cortex is as follows: in the center (fovea) of the visual field  $\frac{1}{7}^\circ$  corresponds with 1 millimeter in the cortex [2]. At the far periphery (the visual field angle near

$8^\circ$ ) 1 millimeter in the cortex corresponds with  $8^\circ$  in the visual field. As one moves from the fovea to the far periphery the relation between cortex and visual field drops logarithmically because its inverted magnification increases in a linear way (3).

Since we are interested in how the visual field is mapped on the cortex, we define the *center of the primary visual cortex* as the position in the primary visual cortex where the center of the visual field is mapped. With this definition we are able to define the relation between the radius from the center of the primary visual cortex and the visual field angle. With a given cortex radius  $\tilde{r}$  (in millimeters) the visual field angle  $\alpha$  is calculated as follows:

$$\alpha(\tilde{r}) = \frac{x_0}{x}(x+1)^{\tilde{r}} - \frac{x_0}{x} \quad (4)$$

where  $x_0 = 0.14$  and  $x = 0.11$  as defined in the linear inverted magnification (3). Equation (4) is derived from the recursive function  $\alpha(n)$ :

$$\alpha(n) = \alpha(n-1) + A(\alpha(n-1)) \quad (5)$$

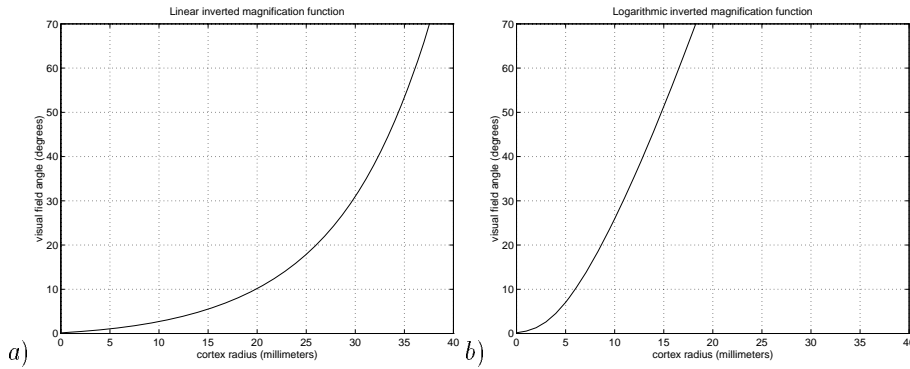
Which is equivalent with:

$$\alpha(n) = \frac{x_0}{x}(x+1)^n - \frac{x_0}{x}$$

If the visual field angle  $\alpha$  is known then the cortex radius  $\tilde{r}$  will be:

$$\tilde{r}(\alpha) = \frac{\log\left(\frac{x}{x_0}\alpha + 1\right)}{\log(x + 1)} \quad (6)$$

From equation 6 and the maximum visual filed angle  $\text{VF}\alpha_{\max}$  we conclude that a radius of about 38 millimeters are needed to cover the whole visual field (Figure 4a). Which in fact is too large, since the the visual cortex has a surface of about 1300-1400  $\text{mm}^2$  [2]. If we assume that the striate cortex has a circular shape than the surface would be about 4600 square millimeters\*. For a correct approximation of the surface of the cortex, not the linear inverted magnification A should be used but the logarithmic inverted magnification L. If L is used only a radius of 19 millimeters is needed to cover the whole visual field (Figure 4b).



**Figure 4:** For a visual field with  $\text{VF}\alpha_{\max} = 70^\circ$ , a radius of 38 millimeters in the cortex is needed to cover the whole visual field when the linear inverted magnification function A is used as can be seen in a). In b) the logarithmic inverted magnification function L is used, a maximum radius of 19 millimeters is needed to cover the whole visual field.

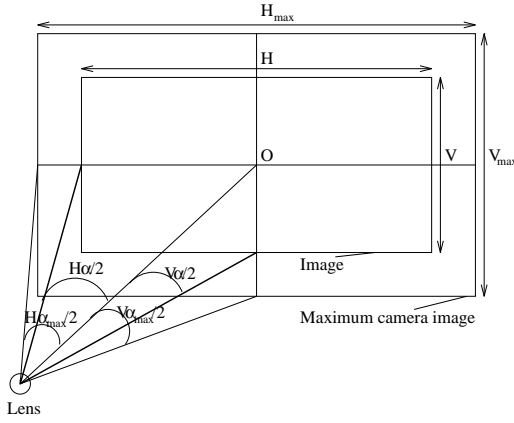
### 3 Physical structure and limits of an artificial two dimensional visual field

If one wants to model a visual system than also an input device (camera) should considered in the model. The setup is usually a camera which is connected to a piece of computer hardware, such that the computer is used to map a three dimensional scene on a discrete two dimensional grid. The so created image can be seen as a **part** of an artificial two dimensional visual field, since the lens used in the camera has a field angle which is smaller than the human visual field angle. The limited visual field is digitized by a frame grabber, which can be seen as the “discrete” map of cones. In the grabber one can cut off a part of the discrete visual field, which makes the visual field even smaller.

Assume that the maximum discrete size of an image created the frame grabber is  $H_{\max} \times V_{\max}$  (the PAL standard usually, for the frame grabber we have it is  $768 \times 579$ ). Define the field angles of the camera lens as  $H\alpha_{\max} \times V\alpha_{\max}$  as maximum constant horizontal field angle and maximum constant vertical field angle respectively (our camera lens has a horizontal and vertical field angle of  $22^\circ 37'$  and  $17^\circ 03'$  respectively).

From the discrete 2-D map of the grabber, we cut out an image  $I(x, y)$  with horizontal and vertical size of H and V respectively. The image I will represent the first neuronal image created in the retina. What should be stressed once more is that with the used lens a maximum field angle of about  $20^\circ$  can be created this in contrast to the human visual system which has a maximum visual field angle of about  $70^\circ$ . The thus created artificial neural image represents only the center of the visual field of a human being.

\*The actually used part in the cortex is much smaller, see section 6 Experimental results.



**Figure 5:** Graphical overview of image sizes and angles.

Given the maximum field angle of the camera-lens and the sizes of the maximum discrete image  $I$ , which can be created by a camera and frame grabber the visual field angle  $\alpha$  of coordinate  $(x, y)$  in image  $I$  can be calculated:

$$\alpha(r) = r \frac{H\alpha_{\max}}{H_{\max}} = r \frac{V\alpha_{\max}}{V_{\max}} \quad (7)$$

and visa versa, given the visual field angle  $\alpha$  and the radius  $r$  to the center of the image:

$$r(\alpha) = \alpha \frac{H_{\max}}{H\alpha_{\max}} \quad (8)$$

where  $r = \sqrt{x^2 + y^2}$ . With the equations (4) and (7) the relation between image and cortex is also defined.

## 4 Receptive fields

Every cell in the visual system has a receptive field, this is an area in the retina where the cell can be influenced. The map of a receptive field, or short receptive field, gives the behavior of the cell. In the visual system a number of different receptive fields are used. Hubel and Wiesel, whose basic techniques have been widely adopted, performed a long series of experiments examining the response characteristics of neurons in the visual cortex. They classified four types of neurons. The first type is the circularly symmetric type (Figure 6a and Figure 6b) which highly reacts on a light spot in a specific position in the visual field. The circular symmetric receptive fields are found in the retina, the lateral geniculate body, and in layer IV<sup>C</sup> of the striate cortex. The second type of receptive fields are the simple cortical cells (Figure 6c and Figure 6d) these cells highly react on a slit of light with a certain orientation and position in the visual field, these filters can be seen as a combination of circular symmetric cells [9]. The third and fourth type are the complex - and hyper-complex receptive fields respectively. There are of course more types of receptive fields but we only consider the first two types of cells here.

### 4.1 Receptive fields weighting functions

In this section the circular symmetric and the simple cortical receptive fields will be modeled by a mathematical function [11]. We use gaussian functions modeling these receptive fields since these functions are “active” within a limited range, which means in fact that in the center of the receptive field the activity is strong and it decreases exponentially if one moves to the periphery of the receptive field. The two different types (circular symmetric and simple) receptive fields are all derived from this function.

The basic two dimensional gaussian function is defined as follows:

$$G(r) = e^{-r^2} \quad (9)$$

The circular symmetric receptive field is a two dimensional function which is called a center-on, surround-off function or because of its shape a mexican-hat function. The mexican-hat function which is derived from (9) can be defined as follows:

$$G_M(r) = \frac{2\sigma - r^2}{\sigma^2} e^{-\frac{r^2}{2\sigma}} \equiv \left\{ r = \sqrt{x^2 + y^2} \right\} \quad G_M(x, y) = \frac{2\sigma - (x^2 + y^2)}{\sigma^2} e^{-\frac{x^2 + y^2}{2\sigma}} \quad (10)$$

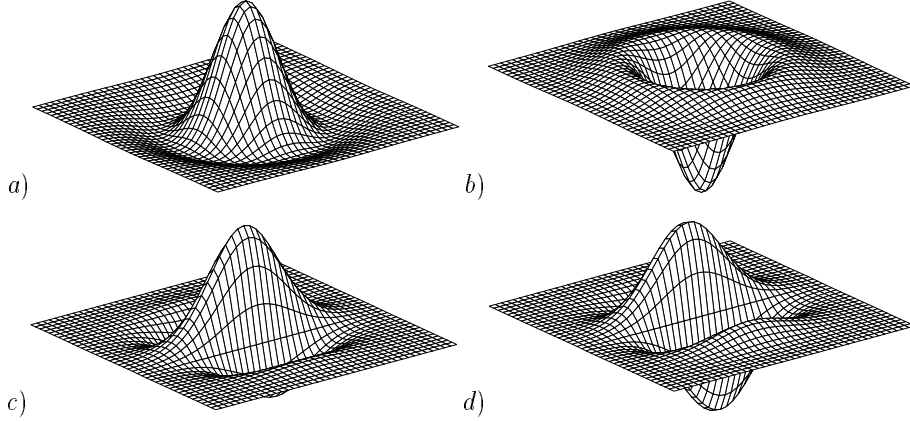
The center-off, surround-on function (Figure 6b) is defined as the negative mexican-hat function  $-G_M(x, y)$ .

The two simple cortical filters, symmetrical orientation sensitive receptive field and anti-symmetrical receptive field can be modeled by the real and imaginary part of a so Gabor function [5, 3].

$$\Re G_{G_\theta}(x, y) = \gamma \cos(\xi) e^{-\frac{x^2+y^2}{2\sigma}} \quad (11)$$

$$\Im G_{G_\theta}(x, y) = \gamma \sin(\xi) e^{-\frac{x^2+y^2}{2\sigma}} \quad (12)$$

where  $\gamma = \frac{3.7534}{2\sigma}$  and  $\xi = \frac{\pi}{\sqrt{2\sigma}}(x \cos \theta + y \sin \theta)$ .



**Figure 6:** Four different receptive field functions. Two ganglion cell functions: a) a center-on surround-off function and b) a center-off surround-on function. Two simple cortical functions, a symmetrical orientation sensitive function and an anti-symmetrical orientation sensitive function are shown in c) and d) respectively.

A common receptive field RF, an arbitrary one from the defined functions (10)-(12), is active within a certain radius  $r$ . Note that the radius can be easily transformed to a visual field (7). If  $R$  is defined as the maximum distance from the center of receptive field RF where RF is still active then the receptive field is inactive if:

$$\text{RF}(R + \Delta) = \varepsilon \quad (13)$$

where  $\varepsilon > 0$  is a constant near 0 where the receptive field is considered as inactive. The used  $\Delta$  is a step size in radius, which is 0 if the visual field is continuous. It should be considered that if one has a discrete visual field not every distance to the center of a receptive field exists, given the radius  $r = \sqrt{x^2 + y^2}$ , the smallest distance larger than  $r$  is  $r + \Delta$ , where

$$r + \Delta = \min \left( \sqrt{(x+1)^2 + y^2}, \sqrt{x^2 + (y+1)^2} \right). \quad (14)$$

This means that  $\Delta$  is:

$$\Delta = \min \left( \sqrt{(x+1)^2 + y^2} - \sqrt{x^2 + y^2}, \sqrt{x^2 + (y+1)^2} - \sqrt{x^2 + y^2} \right) \quad (15)$$

Since the maximum radius  $R + \Delta$  is known,  $\sigma$  in the receptive fields can be derived from the gaussian part  $e^{-\frac{r^2}{2\sigma}}$  in (10)-(12),  $\sigma$  is derived from  $e^{-\frac{(R+\Delta)^2}{2\sigma}} = \varepsilon$ , where  $\varepsilon$  is a small constant greater than 0.

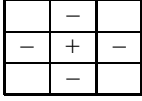
$$\sigma = -\frac{(R + \Delta)^2}{2 \log \varepsilon} \quad (16)$$

The last feature to be described, only concerns the symmetric receptive fields, since they contain a center and a surround. The border between center and surround can be defined as the radius  $b$  in the receptive field where the defined mexican-hat function, defined in (10), will change its sign. Which in formula means that  $G_M(b) = 0$ . Radius  $b$  will be:

$$b = \sqrt{2\sigma} \equiv \left\{ \sigma \text{ from (16)} \right\} \quad b = \frac{R + \Delta}{\sqrt{-\log \varepsilon}} \quad (17)$$

This means that the relation between the receptive field radii of center and surround is  $1 : \sqrt{-\log \varepsilon}$ .

## 4.2 Creating receptive fields in relation to each other



**Figure 7:** Minimal discrete center-on (+), surround-off (-) receptive field.

A receptive field RF is one of the functions as shown in the previous section. The next step is to create a receptive field which size is related to a visual field angle  $\alpha$ . Let  $\rho$  be defined as the receptive field angle. First we have to know the minimum useful receptive field radius. Assume that we have a center-on, surround-off receptive field, then the the minimal discrete receptive field will have, as shown in Figure 7, a discrete radius of one. This means that  $r(\rho(\alpha)) = 1$  and the minimum angle for  $\rho(\alpha)$  will be  $\frac{H_{\alpha_{\max}}}{H_{\max}}$ .

The maximum receptive field angle  $\rho$  should be related to the inverted magnification factor, if one wants to cover the whole visual field. This means that  $\rho$  is proportional to A if the linear inverted magnification function A is used. Thus

$$\rho(\alpha) = cA(\alpha) \quad (18)$$

where c is a constant.

In [6] is described that roughly a 2 millimeter movement in the the cortex is needed to displace fields from one region to an entirely new region. We may conclude that, if one moves two millimeters in the cortex that the receptive fields do not interfere with each other, so the maximum receptive field angle is obtained if fields:

$$\alpha(r) + \rho(\alpha(r)) = \alpha(r+2) - \rho(\alpha(r+2)) \quad \equiv \quad c = \frac{\alpha(r+2) - \alpha(r)}{A(\alpha(r)) + A(\alpha(r+2))} \quad (19)$$

as shown in the left part of Figure 8).

The receptive field angle is known as one moves from the center of the visual field to the far periphery, but the relation between two receptive fields who have the same visual field angle  $\alpha$  is still not defined. So the last step for covering the whole visual field is the relation between two *neighbor receptive fields* who have both the same receptive field angle  $\alpha$  but a different position in the visual field, as shown in the right part of Figure 8. For this relation again a two millimeter movement is used to be in an entire new field region. We assume that two neighbor receptive fields with an equal visual field angle have also a one millimeter position difference in the cortex.

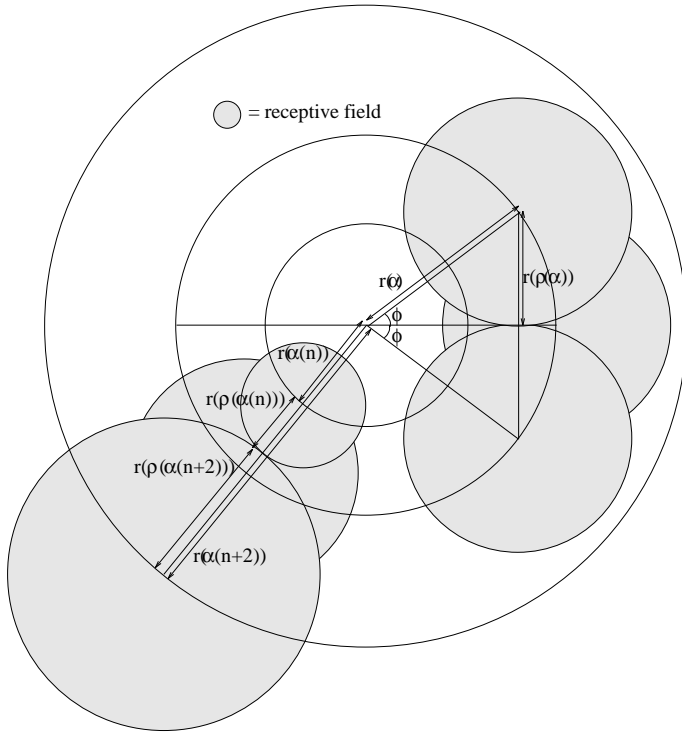
The angle  $\phi$  (Figure 8) needed to cover the visual field for a constant visual field angle  $\alpha$  is as follows:

$$\phi = \arcsin\left(\frac{\rho(\alpha)}{\alpha}\right) \quad (20)$$

The number of receptive fields for a given constant visual field angle  $\alpha$  is:

$$N(\alpha) = \frac{2\pi}{\phi} \quad (21)$$

where N is the number of receptive fields. For a uniform distribution of



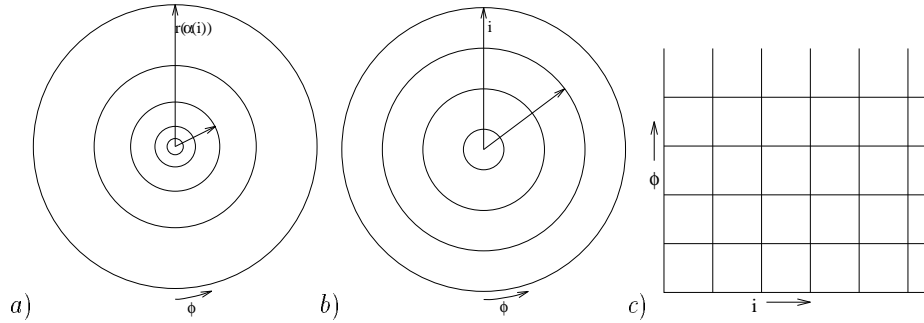
**Figure 8:** The receptive fields are represented by grey circles. By using the neighboring receptive fields in the cortex with a radius of n, n+1, and n+2 millimeters, the maximum receptive field angle  $\rho(\alpha(n))$  can be calculated (left). With a constant visual field angle  $\alpha$  and the receptive field size  $r(\rho(\alpha))$ ,  $\phi$  can be calculated (right).

receptive fields over the visual field, the average number is used for all used visual field angles.



## 5 Modeling a cortical filter

From the properties described in the the previous sections, a receptive field filter can be modeled easily. We call a modeled receptive field filter a *local filter* since the receptive field has a position in the visual field and has a limited activity. A uniform distribution (as described in the previous section) will give the cortical filter.



**Figure 9:** The results of the local filter  $LF_\theta$  can be shown in three different ways: a) with radius  $r$  and orientation  $\phi$ , b) with a relative radius  $i$  and orientation  $\phi$  or c) the relative radius  $i$  and orientation  $\phi$  mapped on a rectangular grid.

A local filter is defined as follows:

$$LF_\theta(r(\alpha(i)), \phi) = \int_{-r(\rho(\alpha(i)))}^{r(\rho(\alpha(i)))} I(x + x_1, y + y_1) RF_{\theta, \sigma}(x_1, y_1) dx_1 dy_1 \quad (22)$$

$$\sigma = -\frac{(r(\rho(\alpha(i))) + \Delta)^2}{2 \log \varepsilon} \quad (23)$$

where  $\sigma$  is derived from equation (16) with  $R = r(\rho(\alpha(i)))$  and  $\Delta = 0$ ,  $i$  is the cortex radius in millimeter,  $I$  is a two-dimensional visual field (image), and  $RF$  is one of the receptive field functions from (10)-(12). The local filter  $LF$  will be displayed as shown in Figure 9a. The local filter should defined slightly different:

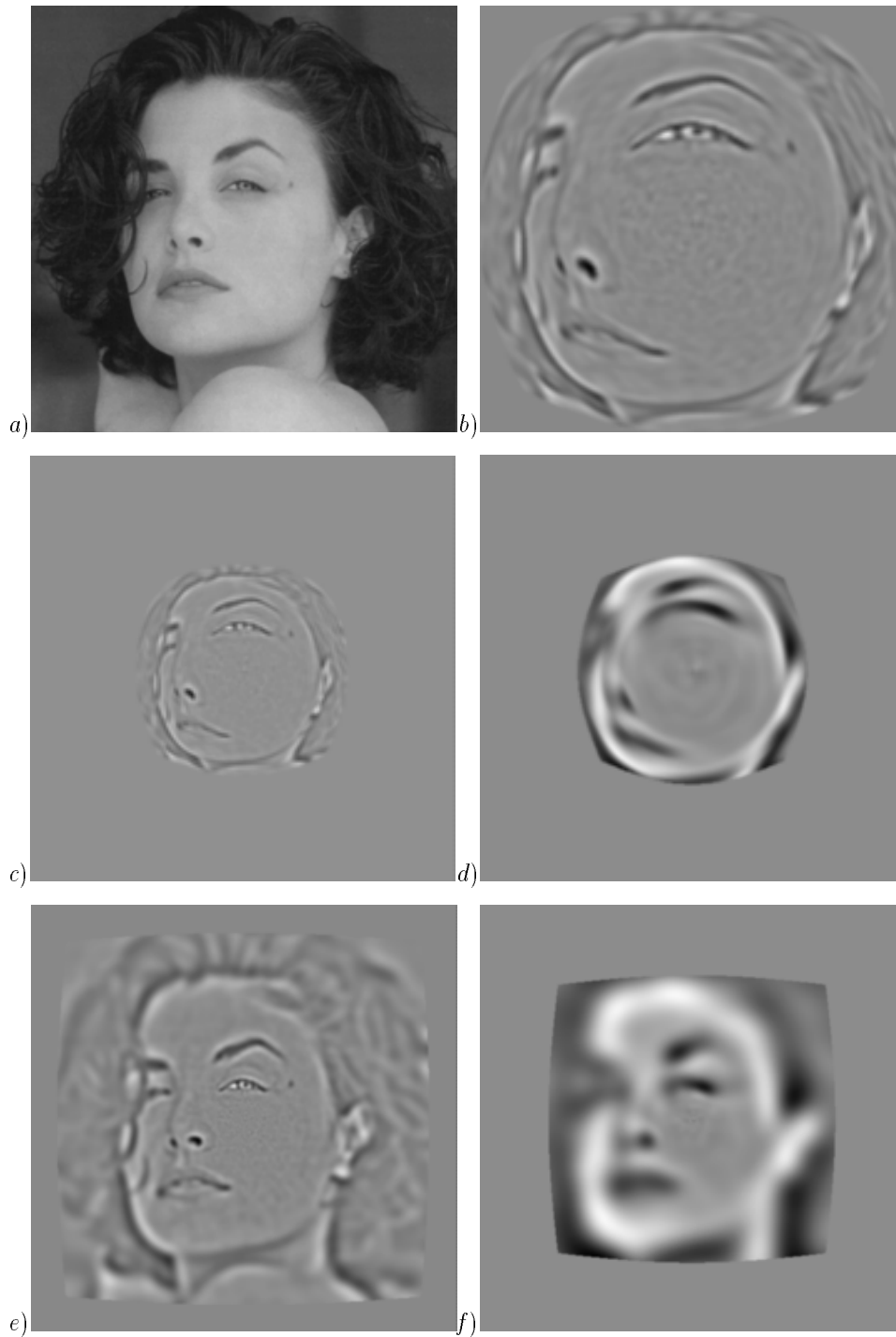
$$LF_\theta(i, \phi) = \int_{-r(\rho(\alpha(i)))}^{r(\rho(\alpha(i)))} I(x + x_1, y + y_1) RF_{\theta, \sigma}(x_1, y_1) dx_1 dy_1 \quad (24)$$

if one wants to use a relative display as shown in Figure 9b and 9c. In the filter  $x = r(\alpha(i)) \cos \phi$ ,  $y = r(\alpha(i)) \sin \phi$ , and  $\theta$  is the orientation of the local filter  $LF_\theta$  if a orientation sensitive (i.e. a simple cortical) receptive field is used, otherwise the orientation  $\theta$  may be ignored. The location of the local filter  $LF_\theta$  in the visual field is given by  $\phi$  and  $r(\alpha(i))$ . The two-dimensional receptive field is represented by a 2-dimensional input image  $I$ .

The cortical filter contains a number of local filters  $LF$  which are distributed over the cortex. If the cortex represents a discrete square grid of a certain step size in millimeters, in every discrete point of the grid a local filter will be calculated.

## 6 Experimental results

Assume that in the center of the visual field a face image is presented, as shown in Figure 10a. In Figure 10e-f we used a combination of center-on surround-off and center-off surround-on cells. The combination is used since negative inputs to neurons will never cause them to fire [14]. The ‘‘ganglion map’’ is not distorted since the ganglion cells do not give a distorted map, this is also the reason that the ganglion cell density decreases with retinal eccentricity [1, 15]. The receptive fields (local filters) are small in the center of the visual field so high detail will be obtained here. The fields grow linear with eccentricity, this means that the



**Figure 10:** A face image which is mapped on the center of the visual field. The exact center of the visual field is a little under the right eye in the image *a*). In *c*) and *d*) the results of a cortical filter using the ganglion type of cells which are used in the striate cortex area  $IV^c$  on a distorted map are shown. A linear inverted magnification factor and a logarithmic inverted magnification factor are used, *c*) and *d*) respectively. An enlargement of *c*) is shown in *b*). In *e*) and *f*) the results of a cortical filter using the ganglion or lateral geniculate cells on a non distorted map are shown. Again for a linear and logarithmic inverted magnification function, *e*) and *f*) respectively.

image will be blurred more and more as one moves away from the center of the visual field. The differences are clear, for example if the two eyes in the filtered images are compared with the eyes in the original image.

The same filter is used for layer IV<sup>C</sup> in the striate cortex, the distorted map will give a completely different view (Figure 10c-d) in comparison with the resulting images from the ganglion cells. In the distorted map there is in comparison a large surface used for the center, where the details are visible. Note that really a small part of the cortex is used from the estimation we made in one of the previous sections. In fact, if the linear inverted magnification function is used the 'effective' radius of the cortex is about 17 millimeters. This in fact means that the effective surface of the cortex is about 1000 square millimeters, if a circular surface is considered. This gives a good approximation of the actual used size in the cortex.

## References

- [1] J. M. Van Buren. *The Retinal Ganglion Cell Layer*. Charles C. Thomas, Springfield, Illinois, U.S.A., 1963.
- [2] P. M. Daniel and D. Whitteridge. The representation of the visual field on the cerebral cortex in monkeys. *Journal of Physiology*, 159:203–221, 1961.
- [3] John G. Daugman. Complete discrete 2-d gabor transforms by neural networks for image analysis and compression. *IEEE Transactions on Acoustics, Speech and Signal Processing*, 36(7):1169–1179, July 1988.
- [4] B. Fischer. Overlap of receptive field centers and representation of the visual field in the cat's optic tract. *Vision Research*, 13:2113–2120, 1973.
- [5] D. Gabor. Theory of communication. *J. Inst. Elec. Eng.*, 93(3):429–457, November 1946.
- [6] David H. Hubel. Exploration of the primary visual cortex, 1955-78. *Nature*, 299:515–524, October 1982.
- [7] David H. Hubel and David C. Freeman. Projection into the visual field of ocular dominance columns in macaque monkey. *Brain Research*, 122:336–343, 1977.
- [8] David H. Hubel and Torsten N. Wiesel. Uniformity of monkey striate cortex: A parallel relationship between field size, scatter, and magnification factor. *Journal of Comparative Neurology*, 158:295–306, 1974.
- [9] D.H. Hubel and T.N. Wiesel. Ferrier Lecture functional architecture of macaque visual cortex. *Proc. R. Soc. Lond. B.*, 198:1–59, July 1977.
- [10] S. W. Kuffler. Discharge patterns and functional organization of mammalian retina. *Journal of Neurophysiology*, 16:37–68, 1953.
- [11] T. Lourens. Modeling retinal high and low contrast sensitivity filters. In J. Mira and F. Sandoval, editors, *Proceedings of the International Workshop on Artificial Neural Networks, IWANN '95*, volume 930 of *Lecture Notes in Computer Science*, pages 61–68, Torremolinos (Málaga), Spain, June 7-9 1995. Springer-Verlag Berlin Heidelberg.
- [12] N. Petkov, P. Kruizinga, and T. Lourens. Biologically motivated approach to face recognition. In J. Mira, J. Cabestany, and A. Prieto, editors, *New Trends in Neural Computation, Proceedings of the International Workshop on Artificial Neural Networks, IWANN '93*, volume 686 of *Lecture Notes in Computer Science*, pages 68–77. Springer-Verlag Berlin Heidelberg, Sitges, Spain, June 9-11 1993.
- [13] N. Petkov and T. Lourens. Human visual systems simulations - an application to face recognition. In H. Dedieu, editor, *Circuit Theory and Design 93, Proceedings of the 11<sup>th</sup> Conference on Circuit Theory and Design*, pages 821–826, Davos, Switzerland, Aug. 30 - Sept. 3 1993. Elsevier Science Publishers B.V. Amsterdam.
- [14] N. Petkov and T. Lourens. Interacting cortical filters for object recognition. In K. Sugihara, editor, *Proceedings of Asian Conference on Computer Vision, ACCV '93*, pages 583–586, Osaka, Japan, Nov. 23-25 1993.
- [15] J. Stone. A quantitative analysis of the distribution of ganglion cells in the cat's retina. *Journal of Comparative Neurology*, 124:337–352, 1965.